Measuring entanglement and entanglement measures

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Abstract: We efficiently produce entangled photon pairs of arbitrary state purity, and describe our investigations into suitable entanglement measures, including the effects of experimental uncertainties. Unlike pure systems, there are several entanglement measures for mixed systems.

Entangled states of multi-particle systems are arguably the quintessential feature of quantum mechanics. In addition to their central role in discussions of non-local quantum correlations, they form the basis of quantum information, and enable such phenomena as quantum cryptography, quantum dense coding, teleportation, and quantum computation.

Via the process of spontaneous parametric downconversion in a pair of non-linear crystals, it is now possible to produce, with high efficiency, pairs of photons entangled in their polarization degree of freedom \cite{1}. Further, we can control both the extent of entanglement \cite{2} and the purity of the state. Density matrices for states generated by this method, and measured using tomographic techniques \cite{2}, are shown in figure 1. Note that even in the best case (Figs 1a and 1b) the experimentally produced states are not completely pure - indeed the states can be smoothly varied up to the completely mixed limit (Fig. 1c). For strictly pure states, the entanglement of the state can be straightforwardly characterized using the entropy of entanglement,

\[
E = -Tr \{ \rho_A \ln(\rho_A) \} = -Tr \{ \rho_B \ln(\rho_B) \},
\]

where \(\rho_A\) (\(\rho_B\)) is the density matrix of the two photon state partially traced over the photon in path A (path B). For mixed states, i.e. those in which the entropy \(S = -Tr \{ \rho \ln(\rho) \} \neq 0\), measures of entanglement are an on-going research problem \cite{3}. An unambiguous measure of entanglement does exist for mixed bi-partite systems \cite{4}, i.e. those consisting of two two-level quantum mechanical sub-systems, such as an entangled photon pair. We demonstrate how this quantity can be determined experimentally, relate it to other measures of entanglement for mixed systems, and consider the effect of experimental uncertainties on all such measures.

References

Fig. 1. Density matrices of entangled photon pairs measured using quantum state tomography. The states are nominally: a) $(|HH\rangle + |VV\rangle)/N_a$; b) $(|HH\rangle + 0.3|VV\rangle)/N_b$; and c) $(|HH\rangle \langle HH| + |HV\rangle \langle HV| + |VH\rangle \langle VH| + |VV\rangle \langle VV|)/N_c$, where $|HH\rangle$ (|VV\rangle) is the quantum state with both photons horizontally (vertically) polarized and $N$ is the appropriate normalization constant.